

Dynamics of cohering and decohering power under Markovian channels

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In this paper, we investigate the cohering and decohering power for the one-qubit Markovian channels with respect to coherence in terms of the l_1 -norm, the Rényi α -relative entropy and the Tsallis α -relative entropy. In the case of $\alpha = 2$, the cohering and decohering power of the amplitude damping channel, the phase damping channel, the depolarizing channel, and the flip channels under the three measures of coherence are calculated analytically. The decohering power on the x, y, z basis referring to the amplitude damping channel, the phase damping channel, the flip channel for every measure we investigated is equal. This property also happens in the cohering power of the phase damping channel, the depolarizing channel, and the flip channels. However, the decohering power of the depolarizing channel is independent to the reference basis, and the cohering power of the amplitude damping channel on the x, y basis is different to that on the z basis.

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I. INTRODUCTION

Coherence, being the heart of interference phenomenon, arising from a critical property which is called quantum superposition. Coherence is one of the most important physical resource in quantum information processing, which can be used in quantum biology [9–11], quantum thermodynamics [12–18], and resource theories [19–26]. The importance of coherence encourages a lot of further studies on this subject [25–35].

Recently, Baumgratz et.al. introduce a rigorous framework for quantifying coherence. A coherence measure satisfies four necessary criterias [1]. Considering a quantum state in a finite dimensional Hilbert space \mathcal{H} with $d = \dim(H)$, we note that \mathcal{I} is a set of incoherent quantum states, which are diagonal in a set of fixed basis $\{|i\rangle\}_{i=1}^d$. Then any proper measure of the coherence C must satisfy the following conditions:

(C1) $C(\rho) \geq 0$ for all quantum states ρ and $C(\rho) = 0$ if and only if $\rho \in \mathcal{I}$;

(C2a) Monotonicity under all the incoherent completely positive and trace-preserving (ICPTP) maps Φ_{ICPTP} : $C(\rho) \geq C(\Phi_{ICPTP}(\rho))$, where $\Phi_{ICPTP}(\rho) = K_n \rho K_n^\dagger$ and K_n is a set of Kraus operators, which satisfies $\sum_n K_n^\dagger K_n \subset \mathcal{I}$ and $K_n \mathcal{I} K_n^\dagger \subset \mathcal{I}$;

(C2b) Monotonicity for average coherence under subselection measurements: $C(\rho) \geq \sum_n p_n C(\rho_n)$ where $\rho_n = \frac{K_n \rho K_n^\dagger}{p_n}$ and $p_n = \text{Tr}(K_n \rho K_n^\dagger)$ for all $\sum_n K_n^\dagger K_n \subset \mathcal{I}$ and $K_n \mathcal{I} K_n^\dagger \subset \mathcal{I}$;

(C3) Non-increasing under mixing of quantum states(convexity): $\sum_n p_n C(\rho_n) \geq C(\sum_n p_n \rho_n)$ for any ensemble $\{p_n, \rho_n\}$.

Moreover, Rastegin et.al. propose the extension condition C2b [3]. The extension condition C2b can be represented as

(C2b') $\sum_n p_n q_n C(\rho) \leq C(\rho)$, where $p_n = \text{Tr}(K_n \rho K_n^\dagger)$, $q_n = \text{Tr}(K_n \delta K_n^\dagger)$, and δ is the nearest incoherent state to ρ .

Considering the properties mentioned above, various of coherence measures have been discussed, such as l_1 -norm of off-diagonal elements of the quantum states [1], the relative entropy of coherence [1], the coherence of formation [24], the Rényi α -relative entropy [5] and the Tsallis α -relative entropy [3].

Quantum coherence can be destroyed by noise in the open system. However, sometimes it can be frozen [19] or increased in some special kinds of channels [7]. To quantifying the power of a channel for creating or destroying the coherence of input quantum states, Mani *et. al.* give the definition of the cohering and the decohering power of quantum channels [2], and Bu *et. al.* define the coherence breaking indices for incoherent quantum channels [36].

In this paper, we mainly study the dynamics of the cohering and the decohering power of Markovian channels under the l_1 -norm, the Rényi α -relative entropy and the Tsallis α -relative entropy. For the difficulty in studying the general value of α , we only consider the case $\alpha = 2$. We find that the decohering power of the amplitude channel, the phase damping channel, and the depolarizing channel under the l_1 -norm, the Rényi 2-relative entropy and the Tsallis

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2-relative entropy are increased monotonically with p from 0 to 1. However, it increases for $p \in [0, \frac{1}{2}]$ and decreases for $p \in [\frac{1}{2}, 1]$ with respect to the flip channels, reaching a maximal value of 1 at $p = \frac{1}{2}$. Moreover, we find the maximum of the cohering power of the amplitude damping channel, the phase damping channel, and the flip channels under the three measures are the same to 1. Finally, we find the cohering power vanishes for the depolarizing channel with respect to arbitrary basis.

This paper is organized as follows. In Sec. II, we introduce the l_1 -norm, the Rényi α -relative entropy and the Tsallis α -relative entropy of coherence. Moreover, we recall the definitions of cohering and decohering power. In Sec. III, we firstly calculate the range of the l_1 -norm, the Rényi α -relative entropy and the Tsallis α -relative entropy. Then we get the expression of that three measures under K basis in the case of $\alpha = 2$. In the end, we study the cohering and decohering power of Markovian channels under the l_1 -norm, the Rényi 2-relative entropy and the Tsallis 2-relative entropy. We summarize our results in Sec. IV.

II. PRELIMINARIES

In order to quantify the cohering and the decohering power of general qubit Markovian channels, we introduce three measurements of coherence, the l_1 -norm, the Rényi α -relative entropy and the Tsallis α -relative entropy.

The l_1 -norm of coherence C_{l_1} is defined as [1]

$$C_{l_1}(\rho) = \min_{\delta \in \mathcal{I}} D(\rho, \delta) = \sum_{i \neq j} |\rho_{ij}|, \quad (1)$$

where $D(\rho, \delta) = \|\rho - \delta\|_1 = \sum_{i,j} |\rho_{ij} - \delta_{ij}|$ denotes the l_1 matrix norm, which means the measure induced by the l_1 -norm is based on the minimal distance of ρ to a set of incoherent states δ . And ρ_{ij} is the off-diagonal element of a quantum state ρ . The l_1 -norm of coherence satisfies the conditions of C1, C2a, C2b and C3 which is proved by Baumgratz et.al., so the l_1 -norm of coherence is a valid coherence measure.

The Rényi α -relative entropy of coherence C_{R_α} is defined as [4, 5]

$$C_{R_\alpha}(\rho) = \min_{\delta \in \mathcal{I}} S_\alpha(\rho \| \delta) = \frac{\alpha}{\alpha - 1} \log \sum_i (\langle i | \rho^\alpha | i \rangle)^{\frac{1}{\alpha}}, \quad (2)$$

where $\alpha \in [0, 2]$. Note that $S_\alpha(\rho \| \delta) = \frac{1}{\alpha - 1} \log \text{Tr}(\rho^\alpha \delta^{1-\alpha})$ for all $0 \leq \alpha$ is the Rényi α -relative entropy, and it reduces to the von Neumann relative entropy when $\alpha \rightarrow 1$, i.e., $\lim_{\alpha \rightarrow 1} S_\alpha(\rho \| \delta) = S(\rho \| \delta) = \text{Tr}[\rho(\ln \rho - \ln \delta)]$. At the same time the C_{R_α} will reduces to the relative entropy of coherence C_r

$$C_r(\rho) = \min_{\delta \in \mathcal{I}} S(\rho \| \delta) = S(\rho_{diag}) - S(\rho). \quad (3)$$

In Ref.[6], Shao *et. al.* show the Rényi α -relative entropy of coherence violate the condition C2b and the extension condition C2b' for $\alpha \in (0, 1)$, so we conclude that the measure of coherence induced by the Rényi α -relative entropy is not a good measure for quantifying coherence. Due to the Rényi α -relative entropy of coherence fulfills the condition C1 and C2a, so it can act as a coherence monotone.

The Tsallis α -relative entropy of coherence C_{T_α} is defined as [3]

$$C_{T_\alpha}(\rho) = \min_{\delta \in \mathcal{I}} D_\alpha(\rho \| \delta) = \frac{1}{\alpha - 1} \left[\left(\sum_i (\langle i | \rho^\alpha | i \rangle)^{\frac{1}{\alpha}} \right)^\alpha - 1 \right], \quad (4)$$

where $D_\alpha(\rho \| \delta) = \frac{\text{Tr}(\rho^\alpha \delta^{1-\alpha}) - 1}{\alpha - 1}$ for $0 < \alpha$ and $\alpha \neq 1$ denotes the Tsallis relative α entropy, and it reduces to the von Neumann relative entropy when $\alpha \rightarrow 1$, i.e., $\lim_{\alpha \rightarrow 1} D_\alpha(\rho \| \delta) = S(\rho \| \delta) = \text{Tr}[\rho(\ln \rho - \ln \delta)]$. At the same time, C_{T_α} will reduces to Eq. (3). In Ref.[3], the author proves that the Tsallis α -relative entropy of coherence satisfies the conditions of C1, C2a and C3 for all $\alpha \in [0, 2]$, but it may violate C2b in some situations. However, it satisfies an extension condition C2b'. We then conclude that the Tsallis α -relative entropy of coherence can be used as a coherence measure.

Mani *et. al.* give the definition of the power of a quantum channel ε for creating or destroying the coherence of input quantum states. The cohering power of a channel is the maximal amount of coherence that it creates when acting on a completely incoherent state. For any quantum channels ε , the cohering power is defined as [2]

$$C_C^K(\varepsilon) = \max_{\rho \in \mathcal{I}} \{C^K(\varepsilon(\rho)) - C^K(\rho)\} = \max_{\rho \in \mathcal{I}} C^K(\varepsilon(\rho)). \quad (5)$$

C_C^K denotes the cohering power of the coherence measure C , K is the reference basis, and we have used property (C1) in the second equation in Eq.(5).

Using the convexity property (C3) of coherence measures, Eq. (5) can be written in a simpler modality on the orthonormal basis vectors

$$C_C^K(\varepsilon) = \max_i C^K(\varepsilon(|k_i\rangle\langle k_i|)). \quad (6)$$

The decohering power of the channel ε is the maximum amount by which it reduces the coherence of a maximally coherent state. The decohering power for a quantum channel ε is define as [2]

$$D_C^K(\varepsilon) = \max_{\rho \in M} \{C^K(\rho) - C^K(\varepsilon(\rho))\}. \quad (7)$$

D_C^K denotes the decohering power of the coherence measure C , and K is the reference basis. M is a set of maximally coherent states. Because all maximally coherent states are pure ones, Eq. (7) can be simplified as

$$D_C^K(\varepsilon) = 1 - \min_{\rho \in M} C^K(\varepsilon(\rho)). \quad (8)$$

Note that the definitions of the cohering and the decohering power of quantum channels are valid for any types of coherence measures.

III. COHERING AND DECOHERING POWER OF MARKOVIAN CHANNELS

As we all know that the coherence of input states may be changed by the quantum channels, then it is necessary to measure the corresponding changes of the quantum coherence. For this purpose, we will study the power of the Markovian noisy one-qubit channels for creating or destorying the quantum coherence. Firstly, we introduce the notion of K coherence, where $\mathbf{k} = (k_x, k_y, k_z)$ is a unit vector standing for the reference K basis $\{\frac{\mathbb{I}+\mathbf{k}\cdot\boldsymbol{\sigma}}{2}, \frac{\mathbb{I}-\mathbf{k}\cdot\boldsymbol{\sigma}}{2}\}$. For a general qubit

$$\rho = \frac{1}{2}(\mathbb{I} + \mathbf{r} \cdot \boldsymbol{\sigma}), \quad (9)$$

where $\mathbf{r} = (r_x, r_y, r_z)$ is a real vector which satisfies $\|\mathbf{r}\| \leq 1$, and $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the vector of Pauli matrices.

Mani *et. al.* give the l_1 -norm of coherence with respect to K basis [2]

$$C_1^K(\rho) = r\sqrt{1 - (\hat{\mathbf{r}} \cdot \mathbf{k})^2}, \quad (10)$$

where $\hat{\mathbf{r}} = \frac{\mathbf{r}}{r}$. In general, for any one-qubit state Eq.(10) satisfies

$$0 \leq C_1^K(\rho) \leq 1. \quad (11)$$

In Ref.[6], Shao *et. al.* give the range of Eq. (2) with respect to a set of basis $\{|i\rangle\}_{i=1}^d$. After simple calculation, we find Eq. (2) satisfies

$$0 \leq C_{R_\alpha}^K(\rho) \leq 1 + \log[2(1 + \|\mathbf{r}\|^2)], \quad (12)$$

for all $\alpha \in [0, 2]$ under K basis. Analogously, we derive the range of Eq. (4) for a general qubit under K basis referring to Ref.[3]

$$0 \leq C_{T_\alpha}^K(\rho) \leq -\ln_\alpha \frac{1}{4(1 + \|\mathbf{r}\|^2)}, \quad (13)$$

for $\alpha \in [0, 2]$, and

$$0 \leq C_{T_\alpha}^K(\rho) \leq \frac{1}{\alpha - 1} \left[4 \left((1 + \|\mathbf{r}\|^2)(1 + \sqrt{4(1 + \|\mathbf{r}\|^2) - 1}) \right)^{\alpha-2} - 1 \right], \quad (14)$$

for $\alpha \in [2, \infty]$, where $\ln_\alpha(x) = \frac{x^{1-\alpha}-1}{1-\alpha}$ is the α logarithm. The maximum value of Eq. (11), Eq. (12), Eq. (13) and Eq. (14) being achieved for a set of maximally coherence pure states with respect to K basis,i.e., $|\psi\rangle = \frac{1}{\sqrt{2}}|\mathbf{k}_+\rangle + e^{i\Omega}|\mathbf{k}_-\rangle$, where $|\mathbf{k}_\pm\rangle$ is eigenvector of $\mathbf{k} \cdot \boldsymbol{\sigma}$.

For the difficulty in calculating the expressions of the Rényi α -relative entropy and the Tsallis α -relative entropy for general α under K basis, we execute a case study of $\alpha = 2$. Substituting Eq. (9) into Eq. (2) and Eq. (4) respectively, we obtain the Rényi 2-relative entropy and the Tsallis 2-relative entropy under K basis as follows

$$C_{R_2}^K(\rho) = 2 \log \left[\frac{1}{2} \left(\sqrt{1 + \|\mathbf{r}\|^2 + 2\mathbf{r} \cdot \mathbf{k}} + \sqrt{1 + \|\mathbf{r}\|^2 - 2\mathbf{r} \cdot \mathbf{k}} \right) \right], \quad (15)$$

$$C_{T_2}^K(\rho) = \left[\frac{1}{2} \left(\sqrt{1 + \|\mathbf{r}\|^2 + 2\mathbf{r} \cdot \mathbf{k}} + \sqrt{1 + \|\mathbf{r}\|^2 - 2\mathbf{r} \cdot \mathbf{k}} \right) \right]^2 - 1. \quad (16)$$

Next, we will study the cohering and decohering power of Markovian noisy one-qubit channels under the l_1 -norm, the Rényi 2-relative entropy and the Tsallis 2-relative entropy in four parts.

A. Amplitude damping channel

To study the cohering and the decohering power of a channel under the l_1 -norm, the Rényi 2-relative entropy and the Tsallis 2-relative entropy, let's start with the amplitude damping channel ε_{ad} , which is characterised by Kraus operators [8]

$$K_1 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}, K_2 = \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix}, \quad (17)$$

where $p \in [0, 1]$ is a parametrization of time t , with $p = 0$ corresponding to $t = 0$ and $p = 1$ corresponding to $t \rightarrow \infty$. It transforms the input Bloch vector $\mathbf{r} = (r_x, r_y, r_z)$ into

$$\varepsilon_{ad}(\mathbf{r}) = (\sqrt{1-p}r_x, \sqrt{1-p}r_y, p + r_z(1-p)). \quad (18)$$

For the decohering power of the amplitude damping channel ε_{ad} , we take $\mathbf{r} \cdot \mathbf{k} = 0$. Then taking Eq. (18) into Eq. (10), Eq. (15) and Eq. (16) respectively, we have the coherence of the maximally coherent states in terms of the l_1 -norm, the Rényi 2-relative entropy and the Tsallis relative 2-entropy

$$C_1^K(\varepsilon_{ad}(\rho)) = \sqrt{\mu - \nu^2}, \quad (19)$$

$$C_{R_2}^K(\varepsilon_{ad}(\rho)) = 2 \log \left[\frac{1}{2} \left(\sqrt{1 + \mu + 2\nu} + \sqrt{1 + \mu - 2\nu} \right) \right], \quad (20)$$

$$C_{T_2,z}^K(\varepsilon_{ad}(\rho)) = \left[\frac{1}{2} \left(\sqrt{1 + \mu + 2\nu} + \sqrt{1 + \mu - 2\nu} \right) \right]^2 - 1, \quad (21)$$

where $\mu = (p^2 - p)r_z^2 + 2p(1-p)r_z + (p^2 - p + 1)$, and $\nu = k_z(r_z\sqrt{1-p}(\sqrt{1-p}-1) + p)$. Note that Eq. (19), Eq. (20), and Eq. (21) are the same for all $\rho \in M$. According to Eq. (8), we have

$$D_1^K(\varepsilon_{ad}) = 1 - \min_{\rho \in M} \sqrt{\mu - \nu^2}, \quad (22)$$

$$D_{R_2}^K(\varepsilon_{ad}) = 1 - \min_{\rho \in M} \left\{ 2 \log \left[\frac{1}{2} \left(\sqrt{1 + \mu + 2\nu} + \sqrt{1 + \mu - 2\nu} \right) \right] \right\}, \quad (23)$$

$$D_{T_2}^K(\varepsilon_{ad}) = 2 - \min_{\rho \in M} \left[\frac{1}{2} \left(\sqrt{1 + \mu + 2\nu} + \sqrt{1 + \mu - 2\nu} \right) \right]^2. \quad (24)$$

For the x, y, z basis, k_z is either 0 or 1, then the decohering power of the amplitude damping channel with respect to x, y, z basis are

$$D_{1,x}^K(\varepsilon_{ad}) = D_{1,y}^K(\varepsilon_{ad}) = D_{1,z}^K(\varepsilon_{ad}) = 1 - \sqrt{1-p}, \quad (25)$$

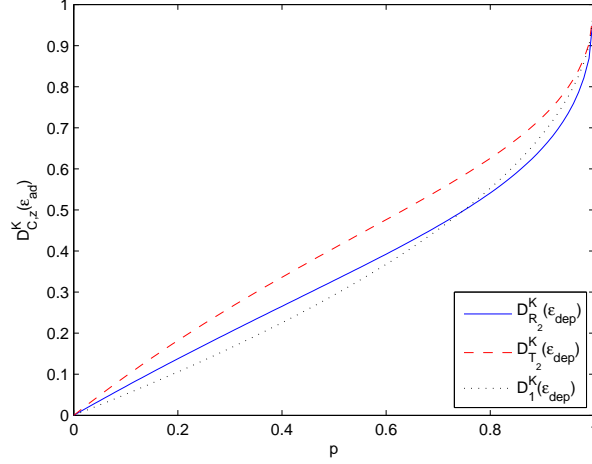


FIG. 1: (Color online) The decohering power of the amplitude damping channel with respect to x, y, z basis. The black points line denotes the decohering power of the amplitude damping channel with respect to the l_1 -norm on the x, y, z basis. The red dotted line denotes the Rényi 2-relative entropy, and the blue valid line denotes the Tsallis 2-relative entropy.

$$D_{R_2,x}^K(\epsilon_{ad}) = D_{R_2,y}^K(\epsilon_{ad}) = D_{R_2,z}^K(\epsilon_{ad}) = 1 - 2 \log \left[\frac{1}{2} \left(\sqrt{p^2 + p + 2} + \sqrt{p^2 - 3p + 2} \right) \right], \quad (26)$$

$$D_{T_2,x}^K(\epsilon_{ad}) = D_{T_2,y}^K(\epsilon_{ad}) = D_{T_2,z}^K(\epsilon_{ad}) = 2 - \left[\frac{1}{2} \left(\sqrt{p^2 + p + 2} + \sqrt{p^2 - 3p + 2} \right) \right]^2. \quad (27)$$

As shown in Fig. 1, we find that the decohering power with respect to the x, y, z basis are increased monotonically with p from 0 to maximal value 1.

In the next, we will calculate the cohering power. In this case, we take $\mathbf{r} = \pm \mathbf{k}$. Taking Eq. (18) into Eq. (10), Eq. (15) and Eq. (16) respectively, we have

$$C_1^K(\epsilon_{ad}(\rho)) = \sqrt{\mu - \omega^2}, \quad (28)$$

$$C_{R_2}^K(\epsilon_{ad}(\rho)) = 2 \log \left[\frac{1}{2} \left(\sqrt{1 + \mu + 2\omega} + \sqrt{1 + \mu - 2\omega} \right) \right], \quad (29)$$

$$C_{T_2,z}^K(\epsilon_{ad}(\rho)) = \left[\frac{1}{2} \left(\sqrt{1 + \mu + 2\omega} + \sqrt{1 + \mu - 2\omega} \right) \right]^2 - 1, \quad (30)$$

where $\omega = k_z^2 \sqrt{1-p}(\sqrt{1-p}-1) + k_z p + \sqrt{1-p}$. According to Eq. (6), we derive the cohering power of the amplitude damping channel under the l_1 -norm, the Rényi 2-relative entropy and the Tsallis 2-relative entropy

$$C_1^K(\epsilon_{ad}) = \sqrt{\mu - \omega^2}, \quad (31)$$

$$C_{R_2}^K(\epsilon_{ad}) = 2 \log \left[\frac{1}{2} \left(\sqrt{1 + \mu + 2\omega} + \sqrt{1 + \mu - 2\omega} \right) \right], \quad (32)$$

$$C_{T_2,z}^K(\epsilon_{ad}) = \left[\frac{1}{2} \left(\sqrt{1 + \mu + 2\omega} + \sqrt{1 + \mu - 2\omega} \right) \right]^2 - 1. \quad (33)$$

The maximal cohering power of the amplitude damping channel with respect to arbitrary basis under every measures studied is 1. For the x, y basis, k_z is 0, we have

$$C_{1,x}^K(\epsilon_{ad}) = C_{1,y}^K(\epsilon_{ad}) = p, \quad (34)$$

$$C_{R_2,x}^K(\varepsilon_{ad}) = C_{R_2,y}^K(\varepsilon_{ad}) = 2 \log \left[\frac{1}{2} \left(\sqrt{p^2 + (\sqrt{1-p} + 1)^2} + \sqrt{p^2 + (\sqrt{1-p} - 1)^2} \right) \right], \quad (35)$$

$$C_{T_2,x}^K(\varepsilon_{ad}) = C_{T_2,y}^K(\varepsilon_{ad}) = \left[\frac{1}{2} \left(\sqrt{p^2 + (\sqrt{1-p} + 1)^2} + \sqrt{p^2 + (\sqrt{1-p} - 1)^2} \right) \right]^2 - 1. \quad (36)$$

The cohering power of the amplitude damping channel on the x, y basis increased monotonically from 0 to 1. For the z basis, k_z is 1. Using Eq. (6) again we have

$$C_{1,z}^K(\varepsilon_{ad}) = C_{R_2,z}^K(\varepsilon_{ad}) = C_{T_2,z}^K(\varepsilon_{ad}) = 0. \quad (37)$$

It means that the amplitude damping channel doesn't have any cohering power with respect to z basis, i.e., the amplitude damping channel ε_{ad} is an incoherent channel on the z basis.

B. Phase damping channel

A quantum channel with Kraus operators

$$K_1 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}, K_2 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{p} \end{pmatrix}, \quad (38)$$

is called phase damping channel [8], denoted by ε_{pd} . It converts the input Bloch vector $\mathbf{r} = (r_x, r_y, r_z)$ into

$$\varepsilon_{pd}(\mathbf{r}) = (\sqrt{1-p}r_x, \sqrt{1-p}r_y, r_z). \quad (39)$$

Substituting Eq. (39) into Eq. (10), Eq. (15), and Eq. (16) respectively, and using Eq. (8) we derive the decohering power of the phase damping channel with respect to the l_1 -norm, the Rényi 2-relative entropy and the Tsallis 2-relative entropy

$$D_1^K(\varepsilon_{pd}) = 1 - \min_{\rho \in M} \sqrt{p(r_z^2 - 1) - (1 - \sqrt{1-p})(r_z k_z)^2 + 1}, \quad (40)$$

$$D_{R_2}^K(\varepsilon_{pd}) = 1 - \min_{\rho \in M} \left\{ 2 \log \left[\frac{1}{2} \left(\sqrt{2-p+pr_z^2+2(1-\sqrt{1-p})r_z k_z} + \sqrt{2-p+pr_z^2-2(1-\sqrt{1-p})r_z k_z} \right) \right] \right\}, \quad (41)$$

$$D_{T_2}^K(\varepsilon_{pd}) = 2 - \min_{\rho \in M} \left[\frac{1}{2} \left(\sqrt{2-p+pr_z^2+2(1-\sqrt{1-p})r_z k_z} + \sqrt{2-p+pr_z^2-2(1-\sqrt{1-p})r_z k_z} \right) \right]^2. \quad (42)$$

After some algebraic calculations, the decohering power of the phase damping channel with respect to the x, y, z basis are

$$D_{1,x}^K(\varepsilon_{pd}) = D_{1,y}^K(\varepsilon_{pd}) = D_{1,z}^K(\varepsilon_{pd}) = 1 - \sqrt{1-p}, \quad (43)$$

$$D_{R_2,x}^K(\varepsilon_{pd}) = D_{R_2,y}^K(\varepsilon_{pd}) = D_{R_2,z}^K(\varepsilon_{pd}) = 1 - \log(2-p), \quad (44)$$

$$D_{T_2,x}^K(\varepsilon_{pd}) = D_{T_2,y}^K(\varepsilon_{pd}) = D_{T_2,z}^K(\varepsilon_{pd}) = p. \quad (45)$$

The change of the decohering power of the phase damping channel with respect to the l_1 -norm, the Rényi 2-relative entropy and the Tsallis 2-relative entropy on the x, y, z basis is in Fig.2(a), from which we can see they all increased monotonically with p from 0 to the maximal value 1.

For the cohering power of the phase damping channel, we take $\mathbf{r} = \pm \mathbf{k}$ as the reference basis which are converted by ε_{pd}

$$\pm (k_x, k_y, k_z) = \pm (\sqrt{1-p}k_x, \sqrt{1-p}k_y, k_z). \quad (46)$$

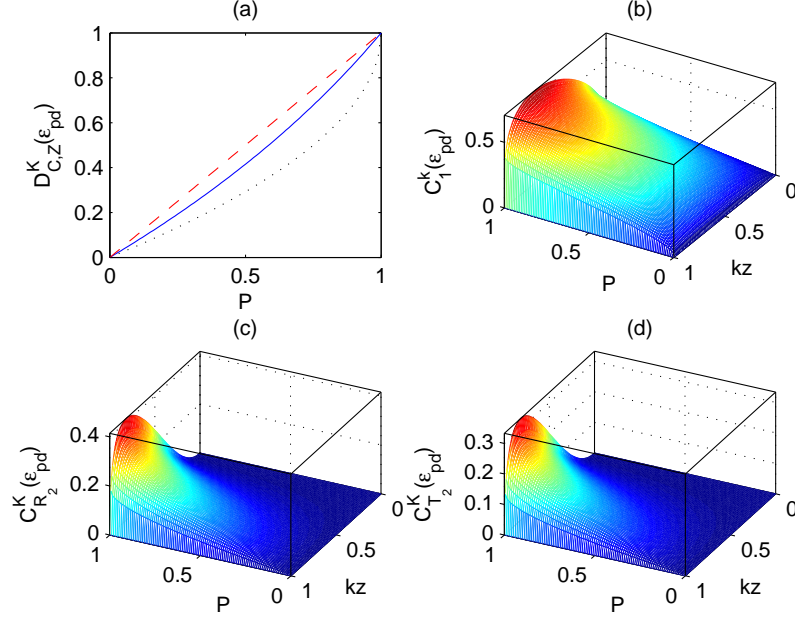


FIG. 2: (Color online) (a) The decohering power of phase damping channel with p under the l_1 -norm(black points line), the Rényi 2-relative entropy(blue valid line), and the Tsallis 2-relative entropy(red dotted line). (b)The cohering power of the phase damping channel under the l_1 -norm, whose maximum is approximately equal to 0.7071. (c)The cohering power the phase damping channel under the Rényi 2-relative entropy, whose maximum is approximately equal to 0.4150. (d)The cohering power of the phase damping channel under the Tsallis 2-relative entropy, whose maximum is approximately equal to 0.3333.

Taking Eq. (46) into Eq. (10), Eq. (15), Eq. (16) respectively, then using Eq. (6) give the cohering power of the phase damping channel with respect to the l_1 -norm, the Rényi 2-relative entropy and the Tsallis 2-relative entropy

$$C_1^K(\varepsilon_{pd}) = \sqrt{p(k_z^2 - 1) - ((\sqrt{1-p} - 1)k_z^2 + \sqrt{1-p})^2 + 1}, \quad (47)$$

$$C_{R_2}^K(\varepsilon_{pd}) = 2 \log \left[\frac{1}{2}(\sqrt{\xi} + \sqrt{\eta}) \right], \quad (48)$$

$$C_{T_2}^K(\varepsilon_{pd}) = \left[\frac{1}{2}(\sqrt{\xi} + \sqrt{\eta}) \right]^2 - 1, \quad (49)$$

where $\xi = k_z^2(p + 2(1 - \sqrt{1-p})) + 2(1 + \sqrt{1-p}) - p$, and $\eta = k_z^2(p - 2(1 - \sqrt{1-p})) + 2(1 - \sqrt{1-p}) - p$. As shown in Fig.2(b), (c) and (d), we have plotted the cohering power of the phase damping channel with respect to arbitrary basis under the three measures. The maximal cohering power of the phase damping channel with respect to the l_1 -norm is 0.7071. While the maximal cohering power of the Rényi 2-relative entropy and the Tsallis 2-relative entropy are 0.4150 and 0.3333, respectively. For the x, y, z basis, k_z is either 0 or 1, so we have

$$C_{1,x}^K(\varepsilon_{pd}) = C_{1,y}^K(\varepsilon_{pd}) = C_{1,z}^K(\varepsilon_{pd}) = 0, \quad (50)$$

$$C_{R_2,x}^K(\varepsilon_{pd}) = C_{R_2,y}^K(\varepsilon_{pd}) = C_{R_2,z}^K(\varepsilon_{pd}) = 0, \quad (51)$$

$$C_{T_2,x}^K(\varepsilon_{pd}) = C_{T_2,y}^K(\varepsilon_{pd}) = C_{T_2,z}^K(\varepsilon_{pd}) = 0. \quad (52)$$

That is to say the phase damping channel has no cohering power on the x, y, z basis.

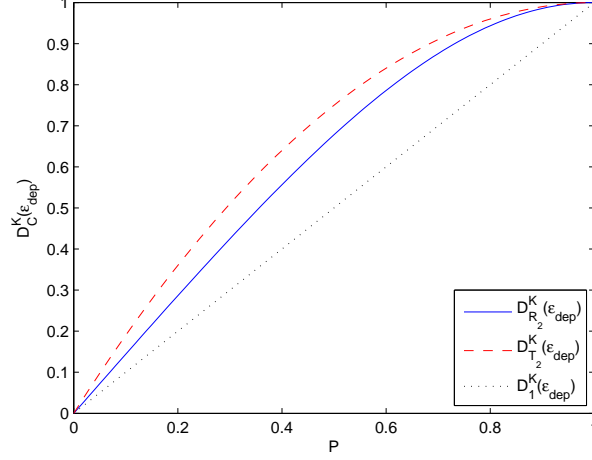


FIG. 3: (Color online) The decohering power of the depolarizing channel with respect to the x, y, z basis. The black points line denotes the decohering power of the depolarizing channel with respect to the l_1 -norm. The red dotted line denotes the Rényi 2-relative entropy, and the blue valid line denotes Tsallis 2-relative entropy.

C. Depolarizing channel

In this section, we will consider a dynamical evolution of a general state ρ under the depolarizing channel ε_{dep} , which is acting as [8]

$$\varepsilon_{dep}(\rho) = (1-p)\rho + p\frac{I}{2}. \quad (53)$$

Taking advantage of Eq. (10), Eq. (15), Eq. (16), and Eq. (8), we have the decohering power of the depolarizing channel

$$D_1^K(\varepsilon_{dep}) = p, \quad (54)$$

$$D_{R_2}^K(\varepsilon_{dep}) = 1 - \log[(1-p)^2 + 1], \quad (55)$$

$$D_{T_2}^K(\varepsilon_{dep}) = 1 - (1-p)^2. \quad (56)$$

The value of the decohering power of the depolarizing channel with respect to the l_1 -norm, the Rényi 2-relative entropy, and the Tsallis relative 2-relative only depend on the parameter p , i.e., the decohering power of the depolarizing channel is same to all reference basis. The variation of the decohering power of the depolarizing channel is depicted in Fig.3

According to Eq. (10), Eq. (15), Eq. (16), and Eq. (6), we have the cohering power of the depolarizing channel with respect to the l_1 -norm, the Rényi 2-relative entropy, and the Tsallis 2-relative entropy

$$C_1^K(\varepsilon_{dep}) = C_{R_2}^K(\varepsilon_{dep}) = C_{T_2}^K(\varepsilon_{dep}) = 0. \quad (57)$$

It is obvious that the depolarizing channel has no cohering power with respect to arbitrary reference basis.

D. Flip channels

Finally, we study the dynamics of the flip channels ε_f^j , which can be described by [8]

$$\varepsilon_f^j(\rho) = (1-p)\rho + p\sigma_j\rho\sigma_j, \quad (58)$$

where $j = x, y, z$ denotes the bit flip channel, bit-phase flip channel and the phase flip channel, respectively.

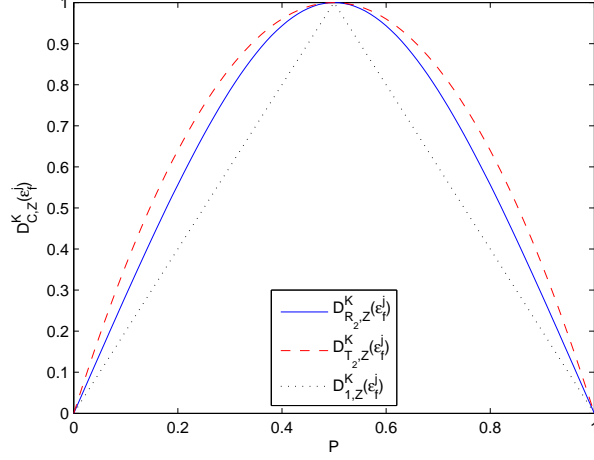


FIG. 4: (Color online) The decohering power of the flip channel on the z basis under the l_1 -norm (black points line), the Rényi 2-relative entropy (blue valid line), and the Tsallis 2-relative entropy (red dotted line denotes).

According to Eq. (10), Eq. (15), Eq. (16) and Eq. (8), the decohering power of the flip channels for the l_1 -norm, the Rényi 2-relative entropy and the Tsallis 2-relative entropy are

$$D_1^K(\epsilon_f^j) = 1 - \min_{\rho \in M} \left\{ \sqrt{4pr_j^2(1 - p(1 - k_j^2)) + (1 - 2p)^2} \right\}, \quad (59)$$

$$D_{R_2}^K(\epsilon_f^j) = 1 - \min_{\rho \in M} \left\{ 2 \log \left[\frac{1}{2} (\sqrt{\tau} + \sqrt{\zeta}) \right] \right\}, \quad (60)$$

$$D_{T_2}^K(\epsilon_f^j) = 2 - \min_{\rho \in M} \left\{ \left[\frac{1}{2} (\sqrt{\tau} + \sqrt{\zeta}) \right]^2 \right\}, \quad (61)$$

where $\tau = 1 + (1 - 2p)^2 + 4pr_j(k_j - (1 - p)r_j)$, and $\zeta = 1 + (1 - 2p)^2 - 4pr_j(k_j + (1 - p)r_j)$. For the x, y, z basis, k_z is either 0 or 1. Then we have, for any $j \in \{x, y, z\}$

$$D_{1,x}^K(\epsilon_f^j) = D_{1,y}^K(\epsilon_f^j) = D_{1,z}^K(\epsilon_f^j) = 1 - |1 - 2p|, \quad (62)$$

$$D_{R_2,x}^K(\epsilon_f^j) = D_{R_2,y}^K(\epsilon_f^j) = D_{R_2,z}^K(\epsilon_f^j) = 1 - \log(1 + (1 - 2p)^2), \quad (63)$$

$$D_{T_2,x}^K(\epsilon_f^j) = D_{T_2,y}^K(\epsilon_f^j) = D_{T_2,z}^K(\epsilon_f^j) = 1 - \log(1 - 2p)^2. \quad (64)$$

The decohering power of flip channels with respect to the x, y, z basis can be seen in Fig. 4, from which we know that they all increased firstly and then decreased, reaching the maximum 1 at $p = \frac{1}{2}$.

Considering the cohering power of the flip channels, we taking Eq.(58) into Eq. (10), Eq. (15), Eq. (16) respectively, then using Eq. (6), we have

$$C_1^K(\epsilon_f^j) = 2pk_j\sqrt{1 - k_j^2}, \quad (65)$$

$$C_{R_2}^K(\epsilon_f^j) = 2 \log \left(\sqrt{(1 - p)^2 + p(2 - p)k_j^2} + p\sqrt{1 - k_j^2} \right), \quad (66)$$

$$C_{T_2}^K(\varepsilon_f^j) = \left[\sqrt{(1-p)^2 + p(2-p)k_j^2} + p\sqrt{1-k_j^2} \right]^2 - 1. \quad (67)$$

The maximal cohering power of the flip channels with respect to the l_1 -norm, the Rényi 2-relative entropy and the Tsallis 2-relative entropy is 1, respectively. For the x, y, z basis, k_j is 0 or 1, then we have, for any $j \in \{x, y, z\}$

$$C_{1,x}^K(\varepsilon_f^j) = C_{1,y}^K(\varepsilon_f^j) = C_{1,z}^K(\varepsilon_f^j) = 0, \quad (68)$$

$$C_{R_2,x}^K(\varepsilon_f^j) = C_{R_2,y}^K(\varepsilon_f^j) = C_{R_2,z}^K(\varepsilon_f^j) = 0, \quad (69)$$

$$C_{T_2,x}^K(\varepsilon_f^j) = C_{T_2,y}^K(\varepsilon_f^j) = C_{T_2,z}^K(\varepsilon_f^j) = 0. \quad (70)$$

That is to say the flip channels doesn't have any cohering power with respect to the x, y, z basis.

IV. CONCLUSION

Quantum coherence plays an important role in quantum information procession. In this paper, We mainly introduce the cohering and decohering power of the Markovian channels in terms of the l_1 -norm, the Rényi α -relative entropy and the Tsallis α -relative entropy. For convenience, we calculate the special case of $\alpha = 2$. We find that the decohering power for the amplitude channel, the phase damping channel, and the depolarizing channel with respect to the three measures we investigated are increased monotonically with p from 0 to 1. But for the flip channels it increases when $p \in [0, \frac{1}{2}]$ and decreases when $p \in [\frac{1}{2}, 1]$, reaching a maximal value of 1 at $p = \frac{1}{2}$. Moreover, the maximal cohering power of the amplitude damping channel and the flip channels are the same to 1. However, the depolarizing channel has no cohering power with respect to arbitrary basis. These results may be useful to the study of coherence.

While we only study the cohering and decohering power of the Markovian channels for one-qubit states with respect to the l_1 -norm, the Rényi α -relative entropy and the Tsallis α -relative entropy in the case of $\alpha = 2$. The cohering and decohering power of one-qubit channels for the general parameters α need further investigated.

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- [1] T. Baumgratz, M. Cramer, and M. B. Plenio: Quantifying Coherence. Phys. Rev. Lett. **113**, 140401(2014).
 - [2] A. Mani and V. Karimipour: Cohering and decohering power of quantum channels. Phys. Rev. A **92**, 032331 (2015).
 - [3] A. E. Rastegin: Quantum-coherence quantifiers based on the Tsallis relative α entropies. Phys. Rev. A **93**, 032136 (2016).
 - [4] E. Chitambar, and G. Gour: Comparison of incoherent operations and measures of coherence. Phys. Rev. A **94**, 052336 (2016).
 - [5] E. Chitambar, and G. Gour: Critical Examination of Incoherent Operations and a Physically Consistent Resource Theory of Quantum Coherence. Phys. Rev. Lett. **117**, 030401 (2016).
 - [6] L. H. Shao, Y. M. Li, Y. Luo, and Z. J. Xi: Quantum coherence quantifiers based on the Rnyi -relative entropy. arXiv:1609.08759 (2016).
 - [7] X. Y. Hu: Channels that do not generate coherence. Phys. Rev. A **94**, 012326 (2016).
 - [8] M. A. Nielsen and I. L. Chuang: Quantum Computation and Quantum Information(Cambridge: Cambridge Univ. Press 2010).
 - [9] M. B. Plenio and S. F. Huelga: Dephasing-assisted transport: quantum networks and biomolecules New J. Phys. **10**, 113019 (2008).
 - [10] S. Lloyd: Quantum coherence in biological systems. J. Phys.: Conf. Ser. **302**, 012037 (2011).
 - [11] F. Levi, and F. Mintert: A quantitative theory of coherent delocalization. New J. Phys. **16**, 033007 (2014).
 - [12] C. A. Rodriguez-Rosario, T. Frauenheim, A. Aspuru-Guzik: Thermodynamics of quantum coherence. arXiv:1308.1245 (2013).
 - [13] M. Lostaglio, D. Jennings, and T. Rudolph: Description of quantum coherence in thermodynamic processes requires constraints beyond free energy. Nat. Commun. **6**, 6383 (2015).
 - [14] M. Lostaglio, D. Jennings, and T. Rudolph: Thermodynamic resource theories, non-commutativity and maximum entropy principles. arXiv:1511.04420 (2015).
 - [15] F. Brandão, M. Horodeckib, N. Ngc, J. Oppenheimc, and S. Wehnerc: The second laws of quantum thermodynamics. Proc. Natl. Acad. Sci. U.S.A. **112**, 3275 (2015).
 - [16] V. Narasimhachar and G. Gour: Low-temperature thermodynamics with quantum coherence. Nat. Commun. **6**, 7689 (2015).
 - [17] P. Cwiklinski, M. Studzinski, M. Horodecki, and J. Oppenheim: Limitations on the Evolution of Quantum Coherences: Towards Fully Quantum Second Laws of Thermodynamics. Phys. Rev. Lett. **115**, 210403 (2015).

- [18] A. Misra, U. Singh, S. Bhattacharya, and A. K. Pati: Energy cost of creating quantum coherence. *Phys. Rev. A* **93**, 052335 (2016).
- [19] T. R. Bromley, M. Cianciaruso, and G. Adesso: Frozen Quantum Coherence. *Phys. Rev. Lett.* **114**, 210401 (2015).
- [20] E. Chitambar and G. Gour: Critical Examination of Incoherent Operations and a Physically Consistent Resource Theory of Quantum Coherence. *Phys. Rev. Lett.* **117**, 030401 (2016).
- [21] D. Girolami: Observable Measure of Quantum Coherence in Finite Dimensional Systems. *Phys. Rev. Lett.* **113**, 170401 (2014).
- [22] C. Napoli, T. R. Bromley, M. Cianciaruso, M. Piani, N. Johnston, and G. Adesso: Robustness of Coherence: An Operational and Observable Measure of Quantum Coherence. *Phys. Rev. Lett. Phys. Rev. Lett.* **116**, 150502 (2016).
- [23] S. Rana, P. Parashar, and M. Lewenstein: Trace-distance measure of coherence. *Phys. Rev. A* **93**, 012110 (2016).
- [24] A. Winter and D. Yang: Operational Resource Theory of Coherence, *Phys. Rev. Lett.* **116**, 120404 (2016).
- [25] S. Du, Z. F. Bai, and Y. Guo: Conditions for coherence transformations under incoherent operations. *Phys. Rev. A* **91**, 052120 (2015).
- [26] L. H. Shao, Z. J. Xi, H. Fan, and Y. M. Li: Fidelity and trace-norm distances for quantifying coherence. *Phys. Rev. A* **91**, 042120(2015).
- [27] C. L. Liu, X. D. Yu, G. F. Xu, and D. M. Tong: Ordering states with coherence measures. *arXiv:1601.03936* (2016).
- [28] Y. Yao, X. Xiao, L. Ge, and C. P. Sun: Quantum coherence in multipartite systems. *Phys. Rev. A* **92**, 022112 (2015).
- [29] S. M. Cheng and Michael J. W. Hall: Complementarity relations for quantum coherence. *Phys. Rev. A* **92**, 042101 (2015).
- [30] M. N. Bera, T. Qureshi, M. A. Siddiqui, and A. K. Pati: Duality of quantum coherence and path distinguishability. *Phys. Rev. A* **92**, 012118 (2015).
- [31] A. Streltsov, U. Singh, H. S. Dhar, M. N. Bera, and G. Adesso: Measuring quantum coherence with entanglement. *Phys. Rev. Lett.* **115**, 020403 (2015).
- [32] Z.J. Xi, M. L. Hu, Y. M. Li, H. Fan: Cohering Power of Unitary Operations and De-cohering of Quantum operations. *arXiv:1510.06473v1* (2015).
- [33] J. X. Chen, S. Grogan, N. Johnston, C. K. Li, S. Plosker: Quantifying the coherence of pure quantum states. *arXiv:1601.06269v2* (2016).
- [34] F. G. Zhang, L. H. Shao, Y. Luo, and Y. M. Li: Ordering states of Tsallis relative -entropies of coherence. *Quantum Information Processing*. DOI: 10.1007/s11128-016-1398-5(2016).
- [35] H. Z. Situ, and X. Y. Hu: Dynamics of relative entropy of coherence under Markovian channels. *Quantum Information Processing*. DOI: 10.1007/s11128-016-1425-6(2016).
- [36] K. F. Bu, Swati, U. Singh, J.D. Wu: Coherence breaking channels and coherence sudden death. *Phys. Rev. A* **94**, 052335 (2016).